Eur. Phys. J. C 48, 605–612 (2006)
DOI 10.1140/epjc/s10052-006-0023-7

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

The effect of the Gaussian profile of the new Higgs doublet on the radiative lepton flavor violating decay

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Received: 20 February 2006 / Revised version: 15 June 2006 /
Published online: 24 October 2006 –

Abstract. We study the branching ratios of the lepton flavor violating processes $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ by considering that the new Higgs scalars localize with Gaussian profile in the extra dimension. We see that the BRs of the LFV decays $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are at the order of magnitude of 10^{-12}, 10^{-16} and 10^{-12} in the considered range of the free parameters. These numerical values are slightly suppressed in the case that the localization points of new Higgs scalars are different from the origin.

1 Introduction

Lepton flavor violating (LFV) interactions have found great interest since they are rich from the theoretical point of view. In the standard model (SM), these decays are allowed by introducing the neutrino mixing with non-zero neutrino masses. However, their branching ratios (BRs) are far below the experimental limits due to the smallness of the neutrino masses. Therefore, they are sensitive to the physics beyond the SM. Since, theoretically, the loop effects are necessary for the existence of LFV decays, it would be possible to predict the free parameters of the underlying theory if one studies the measurable quantities of them.

Among the LFV processes, the radiative LFV $l_i \to l_j\gamma$ ($i \neq j; i,j = e,\mu,\tau$) decays deserve to be analyzed, and there are various experimental and theoretical works in the literature. The current limits for the (BRs) of $\mu \to e\gamma$ and $\tau \to e\gamma$ decays are 1.2×10^{-11} [1] and 3.9×10^{-7} [2], respectively. A new experiment at PSI has been described and aimed to reach a sensitivity of $\mathrm{BR} \sim 10^{-14}$ for $\mu \to e\gamma$ decay [3], and at present the experiment (PSI-R-99-05 Experiment) is still running in the MEG [4]. For $\tau \to \mu\gamma$ decay an upper limit of $\mathrm{BR} = 9.0(6.8)\times 10^{-8}$ at 90% CL has been obtained [5] ([6]), which is an improvement by almost one order of magnitude with respect to the previous one. From the theoretical point of view, there is an extensive work on the radiative LFV decays in the literature [7–21]. In [7–13] these decays were analyzed in the supersymmetric models. [14–19] and [20] were devoted to the radiative LFV decays in the framework of the two Higgs doublet model (2HDM) and in a model independent way, respectively. In another work [21], they are analyzed in the framework of the 2HDM and in the supersymmetric model.

In this work, we study the LFV processes $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ in the 2HDM with the inclusion of a single extra dimension. In the 2HDM the radiative LFV decays are induced by the internal new neutral Higgs bosons h^0 and A^0, and the extension of the Higgs sector results in enhancement in the BRs of these decays. In addition to this, the inclusion of a single extra dimension causes one to modify the BRs. The extra dimension scenario is based on the string theories as a possible solution to the hierarchy problem of the SM. The effects of extra dimensions on various phenomena have been studied in the literature [22–64]. In the extra dimension scenarios the procedure is to pass from higher dimensions to four dimensions by compactifying each extra dimension to a circle S^1 with radius R, which is the typical size of the corresponding extra dimension. This compactification leads to the appearance of new particles, namely Kaluza–Klein (KK) modes in the theory. If all the fields feel the extra dimensions, the so-called universal extra dimensions (UED), the extra dimensional momentum, and therefore the KK number at each vertex, is conserved. If the extra dimensions are accessible to some fields but not all in the theory, they are called non-universal extra dimensions. In this case, the KK number at each vertex is not conserved and tree level interaction of KK modes with the ordinary particles can exist. If the fermions are assumed to be located at different points in the extra dimension with Gaussian profiles, the hierarchy of fermion masses can be obtained from the overlaps of fermion wave functions and such a scenario is called the split fermion scenario [65–81].

Here, we consider that the new Higgs doublet is localized in the extra dimension with a Gaussian profile, by an unknown mechanism; however, the other particle zero modes have uniform profile in the extra dimension. The Higgs localization in the extra dimension has been considered in several works. The work [19] was devoted to the branching ratios of the radiative LFV decays in the

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split fermion scenario, with the assumption that the new Higgs doublet is restricted to the 4D brane or to a part of the bulk in one and two extra dimensions, in the framework of the 2HDM. The idea of the localization of the SM Higgs, using the localizer field, has been studied in [82]. In [83] the new Higgs scalars were localized in the extra dimension, and the SM Higgs was considered to have a constant profile. The localization of new Higgs scalars depended strongly on the strength of the small coupling of the localizer to the new Higgs scalar. In the present analysis, we first consider that the new Higgs scalars localize with Gaussian profiles around the origin in the extra dimension. Second, we assume that the localization point is different from the origin but near to it. We see that the BRs of the LFV decays $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are at the order of magnitude of 10^{-12}, 10^{-16} and 10^{-12} in the given range of the free parameters. These numerical values are slightly suppressed in the case that the localization points of new Higgs scalars are different from the origin.

This paper is organized as follows. In Sect. 2, we present the lepton–lepton–new Higgs scalar vertices and the BRs of the radiative LFV decays with the assumption that the new Higgs doublet is localized with a Gaussian profile in the extra dimension in the 2HDM. Section 3 is devoted to a discussion and our conclusions.

2 The effect of Gaussian profile of new Higgs scalars on the radiative LFV decays in the 2HDM

The radiative LFV $l_i \to l_j\gamma$ decays are rare decays in the sense that they exist at loop level in the SM. Since the numerical values of the BRs of these decays are extremely small in the framework of the SM, one goes to models beyond, where the particle spectrum is extended and the additional contributions result in an enhancement in the numerical values of the physical parameters. Due to the extended Higgs sector, the version of the 2HDM permitting the existence of the FCNCs at tree level, is one of the candidates to obtain relatively large BRs of the decays under consideration. Furthermore, we take into account the effects of the inclusion of a single spatial extra dimension which causes the BRs to enhance due to the fact that the particle spectrum is further extended after the compactification. Here, we consider the effects of the additional Higgs sector with the assumption that the new Higgs scalar zero modes are localized in the extra dimension with Gaussian profiles by an unknown mechanism; on the other hand, the zero modes of the other particles have a uniform profile in the extra dimension. The Yukawa Lagrangian responsible for the LFV interactions in a single extra dimension reads

$$\mathcal{L}_Y = \xi^E_{5ij}\bar{l}_{iL}\phi_2 E_{jR} + \text{h.c.}\,, \tag{1}$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_2 is the new scalar doublet and ξ^E_{5ij} are the FV complex Yukawa couplings in five dimensions, where i, j are family indices of leptons, ϕ_i for $i = 1, 2$, are the two scalar doublets, l_i and E_j are lepton doublets and singlets, respectively. These fields are functions of x^μ and y, where y is the coordinate representing the fifth dimension.

We choose the Higgs doublets ϕ_1 and ϕ_2 as

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ \mathrm{i}\chi^0 \end{pmatrix}\right]\,;$$
$$\phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H^+ \\ H_1 + \mathrm{i}H_2 \end{pmatrix}\,, \tag{2}$$

and their vacuum expectation values read

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}\,; \quad \langle\phi_2\rangle = 0\,. \tag{3}$$

In this case, it is possible to collect the SM (new) particles in the first (second) doublet, and H_1 and H_2 becomes the mass eigenstates h^0 and A^0, respectively, since no mixing occurs between two CP-even neutral bosons H^0 and h^0 at tree level.

The five dimensional lepton doublets and singlets have both chiralities and the four dimensional Lagrangian is constructed by expanding these fields into their KK modes. Besides, the extra dimension denoted by y is compactified on an orbifold S^1/Z_2 with radius R. The KK decompositions of the lepton and the SM Higg fields read

$$\phi_1(x,y) = \frac{1}{\sqrt{2\pi R}}\left\{\phi_1^{(0)}(x) + \sqrt{2}\sum_{n=1}^{\infty}\phi_1^{(n)}(x)\cos(ny/R)\right\}\,,$$
$$l_i(x,y) = \frac{1}{\sqrt{2\pi R}}\left\{l_{iL}^{(0)}(x)\right.$$
$$\left. + \sqrt{2}\sum_{n=1}^{\infty}\left[l_{iL}^{(n)}(x)\cos(ny/R) + l_{iR}^{(n)}(x)\sin(ny/R)\right]\right\}\,,$$
$$E_i(x,y) = \frac{1}{\sqrt{2\pi R}}\left\{E_{iR}^{(0)}(x)\right.$$
$$\left. + \sqrt{2}\sum_{n=1}^{\infty}\left[E_{iR}^{(n)}(x)\cos(ny/R) + E_{iL}^{(n)}(x)\sin(ny/R)\right]\right\}\,, \tag{4}$$

where $\phi_1^{(0)}(x)$, $l_{iL}^{(0)}(x)$ and $E_{iR}^{(0)}(x)$ are the four dimensional Higgs doublet, lepton doublets and lepton singlets respectively. Here, we assume that the new Higgs scalars are localized in the extra dimension with Gaussian profiles,

$$S(x,y) = A e^{-\beta y^2} S(x)\,, \tag{5}$$

by an unknown mechanism[1]. The normalization constant A is

[1] We consider the zero mode Higgs scalars and we do not take into account the possible KK modes of Higgs scalars, since the mechanism for the localization is unknown and we expect that those contributions are small due to their heavy masses.

$$A = \frac{(2\beta)^{1/4}}{\pi^{1/4}\sqrt{\mathrm{Erf}\left[\sqrt{2\beta}\pi R\right]}}\,, \tag{6}$$

and the parameter $\beta = 1/\sigma^2$ regulates the amount of localization, where $\sigma = \rho R$, is the Gaussian width of $S(x, y)$ in the extra dimension. Here the function Erf$[z]$ is the error function, which is defined by

$$\mathrm{Erf}\,[z] = \frac{2}{\sqrt{\pi}}\int_0^z \mathrm{e}^{-t^2}\,\mathrm{d}t\,. \tag{7}$$

The coupling of the new Higgs doublet to the leptons brings about modified Yukawa interactions in four dimensions. To obtain the lepton–lepton–Higgs interaction coupling in four dimensions we need to integrate the combination $\bar{f}^{(0(n))}_{iL(R)}(x, y)S(x, y)f^{(n(0))}_{jR(L)}(x, y)$, appearing in the part of the Lagrangian (1), over the fifth dimension. Using the KK basis for lepton fields (see (4)), we get

$$\begin{aligned}\int_{-\pi R}^{\pi R}&\mathrm{d}y\bar{f}^{(0(n))}_{iL(R)}(x, y)S(x, y)f^{(n(0))}_{jR(L)}(x, y)\\ &= V_n\bar{f}^{(0(n))}_{iL(R)}(x)S(x)f^{(n(0))}_{jR(L)}(x)\,,\end{aligned} \tag{8}$$

where the factor V_n reads

$$V_n = Ac_n\,, \tag{9}$$

and the function A is defined in (6). Here, the fields $f^{(n(0))}_{iL}$, $f^{(n(0))}_{iR}$ are four dimensional left and right handed zero (n) mode lepton fields. The function c_n in (9) is obtained:

$$c_n = \mathrm{e}^{-\frac{n^2}{4\beta R^2}}\frac{\left(\mathrm{Erf}\left[\frac{\mathrm{i}n+2\beta\pi R^2}{2\sqrt{\beta}R}\right]+\mathrm{Erf}\left[\frac{-\mathrm{i}n+2\beta\pi R^2}{2\sqrt{\beta}R}\right]\right)}{4\sqrt{\beta\pi}R}\,. \tag{10}$$

Notice that the Yukawa couplings ξ^E_{ij} in four dimensions are

$$\xi^E_{ij} = A\xi^E_{5ij}\,, \tag{11}$$

where ξ^E_{5ij} are the Yukawa couplings in five dimensions (see (1))[2].

Now, we consider that the new Higgs scalars are localized in the extra dimension at the point y_H, $y_\mathrm{H} = \alpha R$ near the origin, namely,

$$S(x, y) = A_\mathrm{H}\mathrm{e}^{-\beta(y-y_\mathrm{H})^2}S(x)\,, \tag{12}$$

with the normalization constant

$$A_\mathrm{H} = \frac{2(\beta)^{1/4}}{(2\pi)^{1/4}\sqrt{\mathrm{Erf}\left[\sqrt{2\beta}(\pi R+y_\mathrm{H})\right]+\mathrm{Erf}[\sqrt{2\beta}(\pi R-y_\mathrm{H})]}}\,. \tag{13}$$

[2] In the following we use the dimensionful coupling $\bar{\xi}^E_{\mathrm{N},ij}$ in four dimensions, with the definition $\xi^E_{\mathrm{N},ij} = \sqrt{\frac{4G_\mathrm{F}}{\sqrt{2}}}\bar{\xi}^E_{\mathrm{N},ij}$, where N denotes the word "neutral".

After integrating the combination $\bar{f}^{(0(n))}_{iL(R)}(x, y)S(x, y)$ $f^{(n(0))}_{jR(L)}(x, y)$ over the extra dimension, the factor V_n in (8) reads

$$V_n = A_\mathrm{H}c_n\,, \tag{14}$$

with the function A_H in (13). The function c_n in (14) is calculated to be

$$\begin{aligned}c_n &= \mathrm{e}^{-\frac{n^2}{4\beta R^2}}\cos\left[\frac{ny_\mathrm{H}}{R}\right]\\ &\times\frac{\left(\mathrm{Erf}\left[\frac{\mathrm{i}n+2\beta\pi R^2}{2\sqrt{\beta}R}\right]+\mathrm{Erf}\left[\frac{-\mathrm{i}n+2\beta\pi R^2}{2\sqrt{\beta}R}\right]\right)}{4\sqrt{\beta\pi}R}\,.\end{aligned} \tag{15}$$

Similar to the previous case, we define the Yukawa couplings in four dimensions by

$$\xi^E_{ij} = A_\mathrm{H}\xi^E_{5ij}\,. \tag{16}$$

Now, we will present the decay widths of the LFV processes $\mu\to e\gamma$, $\tau\to e\gamma$ and $\tau\to\mu\gamma$. These decays exist at least at one loop level in the 2HDM, and the logarithmic divergences appear in the calculations. These divergences can be eliminated by using the on-shell renormalization scheme[3]. The decay width Γ for the $l_i\to l_j\gamma$ decay reads

$$\Gamma(l_i\to l_j\gamma) = c_1(|A_1|^2+|A_2|^2)\,, \tag{17}$$

for $l_i(l_j) = \tau;\mu(\mu$ or $e;e)$. Here

$$c_1 = \frac{G_\mathrm{F}^2\alpha_\mathrm{em}m^3_{l_i}}{32\pi^4}\,,$$

A_1 (A_2) is the left (right) chiral amplitude and taking only τ lepton for the internal line[4], they read

$$\begin{aligned}A_1 = Q_\tau\frac{1}{48m_\tau^2}&\Bigg\{6m_\tau\bar{\xi}^{E*}_{\mathrm{N},\tau f_2}\bar{\xi}^{E*}_{\mathrm{N},f_1\tau}\Bigg(c_0^2(F(v_{h^0})-F(v_{A^0}))\\ &+2\sum_{n=1}^{\infty}\frac{m_\tau}{\sqrt{m_\tau^2+m_n^2}}c_n^2\Big(F(v_{n,h^0})-F(v_{n,A^0})\Big)\Bigg)\\ &+m_{f_1}\bar{\xi}^{E*}_{\mathrm{N},\tau f_2}\bar{\xi}^{E}_{\mathrm{N},\tau f_1}\Bigg(c_0^2\Big(G(v_{h^0})+G(v_{A^0})\Big)\\ &+2\sum_{n=1}^{\infty}\frac{m_\tau^2}{m_\tau^2+m_n^2}c_n^2(G(v_{n,h^0})+G(v_{n,A^0}))\Bigg)\Bigg\}\,,\end{aligned}$$

[3] In this scheme, the self-energy diagrams for on-shell leptons vanish, since they can be written as $\sum(p) = (\hat{p}-m_{l_1})\bar{\sum}(p)(\hat{p}-m_{l_2})$; however, the vertex diagrams (see Fig. 1) give a non-zero contribution. In this case, the divergences can be eliminated by introducing a counter term V^C_μ with the relation $V^\mathrm{Ren}_\mu = V^0_\mu + V^\mathrm{C}_\mu$, where V^Ren_μ (V^0_μ) is the renormalized (bare) vertex and by using the gauge invariance $k^\mu V^\mathrm{Ren}_\mu = 0$. Here, k^μ is the four momentum vector of the outgoing photon.

[4] We take into account only the internal τ lepton contribution, since we respect the Sher scenario [84]; results in the couplings $\bar{\xi}^E_{\mathrm{N},ij}$ ($i, j = e, \mu$) are small compared to $\bar{\xi}^E_{\mathrm{N},\tau i}$ ($i = e, \mu, \tau$) due to the possible proportionality of them to the masses of leptons under consideration in the vertices.

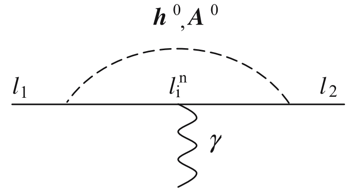

Fig. 1. One loop diagrams contribute to $l_1 \to l_2\gamma$ decay due to the zero mode neutral Higgs bosons h^0 and A^0 in the 2HDM, for a single extra dimension. Here l_i^n represents the internal KK mode charged lepton and $n = 0, 1, \ldots$

$$
\begin{aligned}
A_2 = Q_\tau \frac{1}{48 m_\tau^2} \Bigg\{ & 6 m_\tau \bar{\xi}^E_{N,f_2\tau} \bar{\xi}^E_{N,\tau f_1} \Bigg(c_0^2 \Big(F(v_{h^0}) - F(v_{A^0}) \Big) \\
& + 2 \sum_{n=1}^{\infty} c_n^2 \frac{m_\tau}{\sqrt{m_\tau^2 + m_n^2}} \Big(F(v_{n,h^0}) - F(v_{n,A^0}) \Big) \Bigg) \\
& + m_{f_1} \bar{\xi}^E_{N,f_2\tau} \bar{\xi}^{E*}_{N,f_1\tau} \Bigg(c_0^2 \Big(G(v_{h^0}) + G(v_{A^0}) \Big) \\
& + 2 \sum_{n=1}^{\infty} c_n^2 \frac{m_\tau^2}{m_\tau^2 + m_n^2} \Big(G(v_{n,h^0}) + G(v_{n,A^0}) \Big) \Bigg) \Bigg\} ,
\end{aligned} \tag{18}
$$

where $v_{n,S} = \frac{m_\tau^2 + m_n^2}{m_S^2}$, $m_n = \frac{n}{R}$ and Q_τ is the charge of τ lepton. Here the vertex factor c_n is defined in (10) and (15), and the functions $F(w)$ and $G(w)$ are

$$
\begin{aligned}
F(w) &= \frac{w(3 - 4w + w^2 + 2 l n w)}{(-1+w)^3} , \\
G(w) &= \frac{w(2 + 3w - 6w^2 + w^3 + 6 w l n w)}{(-1+w)^4} .
\end{aligned} \tag{19}
$$

3 Discussion

In the present work, we study the radiative LFV decays $l_i \to l_j\gamma$ in the 2HDM with the addition of a single spatial extra dimension. Here, we consider that new Higgs scalars are localized in the extra dimension with Gaussian profiles by an unknown mechanism; on the other hand, the other particles have uniform zero mode profiles in the extra dimension which is compactified onto orbifold S_1/Z_2. Since these decays exist at least at one loop level, there appear free parameters related to the model used in the theoretical values of the physical quantities. The Yukawa couplings $\bar{\xi}^E_{N,ij}$, $i, j = e, \mu, \tau$, are among those parameters. We consider that the couplings $\bar{\xi}^E_{N,ij}$, $i, j = e, \mu$ are smaller compared to $\bar{\xi}^E_{N,\tau i}$, $i = e, \mu, \tau$, since the latter ones contain heavy flavor. Furthermore, we assume that, in four dimensions, the couplings $\bar{\xi}^E_{N,ij}$ are symmetric with respect to the indices i and j. For the decays under consideration the Yukawa couplings $\bar{\xi}^E_{N,\tau e}$, $\bar{\xi}^E_{N,\tau\mu}$ and $\bar{\xi}^E_{N,\tau\tau}$ play the main role. The Yukawa coupling $\bar{\xi}^E_{N,\tau\mu}$ has been restricted by using the experimental uncertainty, 10^{-9}, in the measurement of the muon anomalous magnetic moment and the upper limit of $\bar{\xi}^E_{N,\tau\mu}$ is predicted near 50 GeV (see [85] and references therein) without extra dimension effects. In our numerical analysis we take its magnitude around this quantity. For the coupling $\bar{\xi}^E_{N,\tau e}$ ($\bar{\xi}^E_{N,\tau\mu}$) we choose the numerical values as smaller (larger) compared to $\bar{\xi}^E_{N,\tau\mu}$, respecting the experimental measurements of the BRs of these decays. For the Higgs masses we take the numerical values $m_{h^0} = 100$ GeV, $m_{A^0} = 200$ GeV.

The compactification scale $1/R$ and the Gaussian width σ of the new Higgs doublet are the additional free parameters which are chosen not to contradict with the experimental results. The direct limits from searching for KK gauge bosons imply $1/R > 800$ GeV; the precision electro weak bounds on higher dimensional operators generated by KK exchange place a far more stringent limit $1/R > 3.0$ TeV [86] and, from $B \to \phi K_S$, the lower bounds for the scale $1/R$ have been obtained as $1/R > 1.0$ TeV, while from $B \to \psi K_S$ one got $1/R > 500$ GeV, and from the upper limit of the BR, $\mathrm{BR}(B_s \to \mu^+\mu^-) < 2.6 \times 10^{-6}$, the estimated limit was $1/R > 800$ GeV [69]. Here, we take the compactification scale $1/R$ in the range $200\,\mathrm{GeV} \le 1/R \le 1000$ GeV and choose the Gaussian width at most at $\rho = 0.05$.

Our analysis is based on the Higgs localization width and the compactification scale dependence of the BRs of the LFV decays. First, we consider that the new Higgs is localized around the origin in the extra dimension. Furthermore, we choose the localization point near the origin and study its effect on the BRs. In our calculations we observe that there is a strong sensitivity of the BRs of the LFV decays to the localization width of the new Higgs doublet in the extra dimension and that it is due to the present 5D construction [5].

Figure 2 represents the $\mathrm{BR}(\mu \to e\gamma)$ with respect to ρ, for different values of the Yukawa couplings $\bar{\xi}^E_{N,\tau\mu}$ and $\bar{\xi}^E_{N,\tau e}$. Here the lower–upper solid (dashed) line represents the BR for a single extra dimension without–with lepton KK modes, the real couplings being $\bar{\xi}^E_{N,\tau\mu} = 50$ GeV, $\bar{\xi}^E_{N,\tau e} = 0.1$ GeV ($\bar{\xi}^E_{N,\tau e} = 0.5$ GeV) and $R = 0.005\,\mathrm{GeV}^{-1}$. It is observed that the BR is strongly sensitive to the Gaussian width of the localized neutral Higgs scalars, and it increases with the increasing values of the width. This enhancement is almost at the order of 10^4 in the range $0.005 \le \rho \le 0.05$. The inclusion of lepton KK modes brings about an additional enhancement at one order. The BR is at the order of the magnitude of 10^{-14} (10^{-12}) for the

[5] The normalization constant A of the Gaussian function, describing the localization of the new Higgs scalar (see (5)), is embedded into the Yukawa couplings ξ^E_{ij} in four dimensions as given in (11) and the couplings ξ^E_{ij} are taken fixed. On the other hand the parameter c_0 appearing in the vertex factor V_n (see (9)) is sensitive to the localization width σ and increases with its increasing values. This picture will not be affected if one considers the parameter c_n, $n \ge 1$, since it contains an exponential suppression factor. Notice that c_0 reaches unity when $\sigma \to \infty$ and this is the case where the Higgs zero mode profile in the extra dimension is uniform.

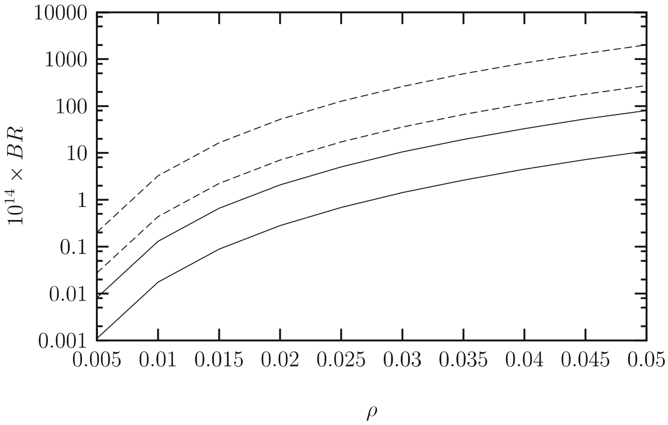

Fig. 2. The BR($\mu \to e\gamma$) with respect to ρ. Here the *lower-upper solid* (*dashed*) *line* represents the BR for a single extra dimension without–with lepton KK modes, the real couplings being $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 50$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau e} = 0.5$ GeV) and $R = 0.005\ \mathrm{GeV}^{-1}$

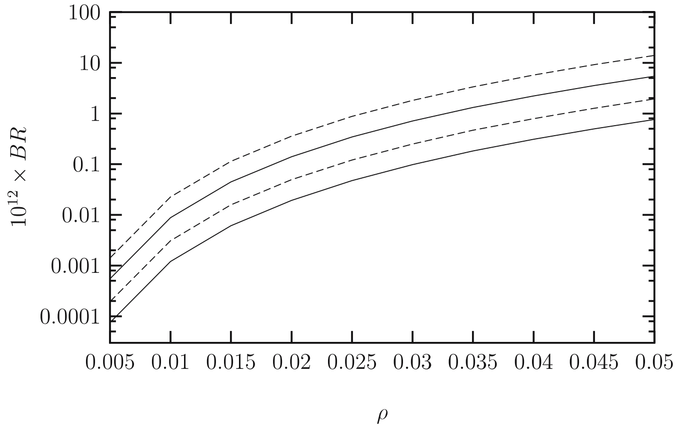

Fig. 4. The BR($\tau \to \mu\gamma$) with respect to ρ. Here the *lower–upper solid (dashed) line* represents the BR for a single extra dimension without–with lepton KK modes, for $R = 0.005\ \mathrm{GeV}^{-1}$, the real couplings being $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 50$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau\mu} = 80$ GeV)

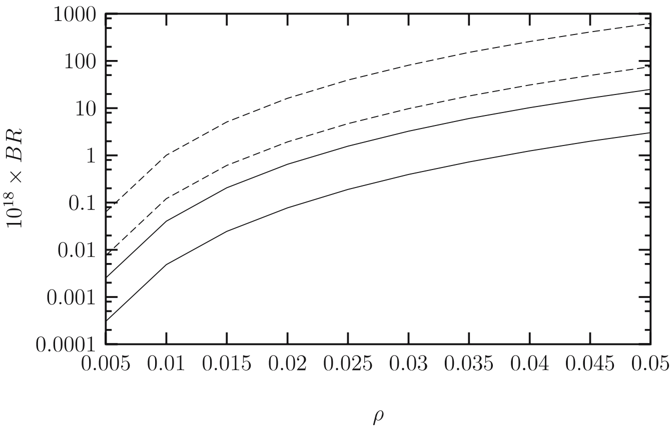

Fig. 3. The BR($\tau \to e\gamma$) with respect to ρ. Here the *lower–upper solid (dashed) line* represents the BR for a single extra dimension without–with lepton KK modes, for $R = 0.005\ \mathrm{GeV}^{-1}$, the real couplings being $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau e} = 0.5$ GeV)

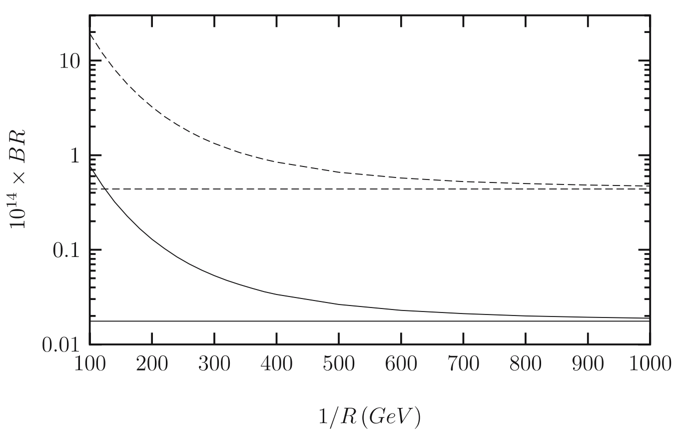

Fig. 5. The BR($\mu \to e\gamma$) with respect to the compactification scale $1/R$. Here the *lower–upper solid (dashed) line* represents the BR for a single extra dimension without–with lepton KK modes, the real couplings being $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 50$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau e} = 0.5$ GeV) and $\rho = 0.01$

intermediate values of the localization parameter ρ and the coupling $\bar{\xi}^E_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau e} = 0.5$ GeV). A new experiment at PSI has been described which aims to reach a sensitivity of BR $\sim 10^{-14}$, and at present the experiment (PSI-R-99-05 Experiment) is still running in the MEG [4]. The improvement of the numerical result of the BR would make it possible to search for the effects of the extra dimensions and the possible localization of new Higgs bosons in the extra dimension.

In Figs. 3 and 4 we present the BR($\tau \to e\gamma$) and BR($\tau \to \mu\gamma$) with respect to ρ, for different values of the Yukawa couplings. Here the lower–upper solid (dashed), line represents the BR for a single extra dimension without–with lepton KK modes, for $R = 0.005\ \mathrm{GeV}^{-1}$, the real couplings being $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau e} = 0.5$ GeV) and $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 50$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau\mu} = 80$ GeV). It is observed that the BR is sensitive to the Gaussian width of the localized neutral Higgs scalars and there is an enhancement at the order of 10^4 in the range $0.005 \le \rho \le 0.05$ similar to the previous decay. The BR is at the order of the magnitude of 10^{-18} (10^{-16}) and 10^{-13} (10^{-12}) for the intermediate values of the localization parameter ρ and the coupling $\bar{\xi}^E_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau e} = 0.5$ GeV) and $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 50$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau\mu} = 80$ GeV). Notice that the inclusion of lepton KK modes brings in an additional enhancement at order one, in both decays.

Figures 5–7 are devoted to the BR($\mu \to e\gamma$); BR($\tau \to e\gamma$) and BR($\tau \to \mu\gamma$), with respect to the compactification scale $1/R$. Here the lower–upper solid (dashed) line represents the BR for a single extra dimension without–with lepton KK modes, the real couplings being $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 50$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^E_{\mathrm{N},\tau e} = 0.5$ GeV); $\bar{\xi}^E_{\mathrm{N},\tau\tau} = 100$ GeV,

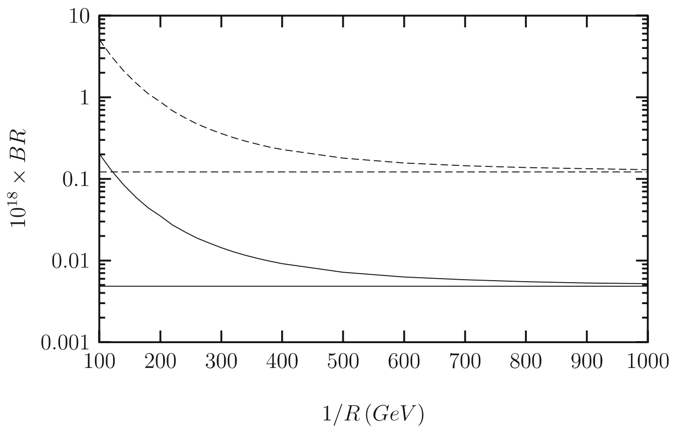

Fig. 6. BR($\tau \to e\gamma$) with respect to the compactification scale $1/R$. Here the *lower–upper solid (dashed) line* represents the BR for a single extra dimension without–with lepton KK modes, the real couplings being $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.5$ GeV) and $\rho = 0.01$

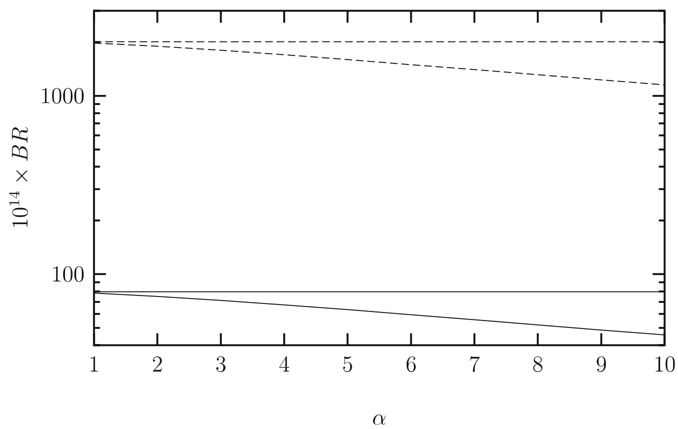

Fig. 8. The BR($\mu \to e\gamma$) with respect to α. Here the *lower–upper solid (dashed) line* represents the BR for a single extra dimension with lepton KK modes, for $y_{\mathrm{H}} = 0 - y_{\mathrm{H}} = \alpha\sigma$, the real couplings being $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 50$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.5$ GeV) and $R = 0.005\,\mathrm{GeV}^{-1}$

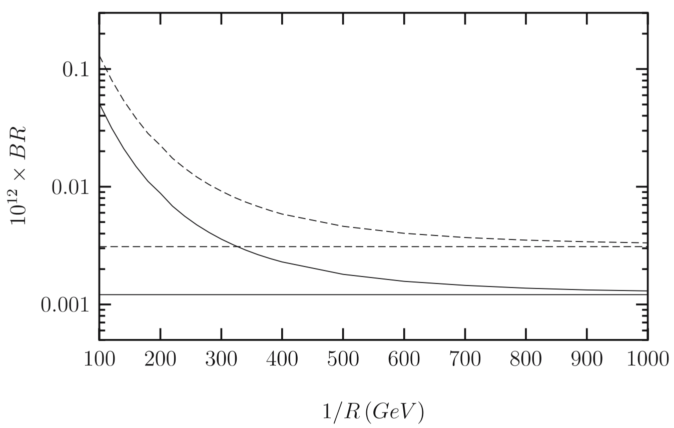

Fig. 7. BR($\tau \to \mu\gamma$) with respect to the compactification scale $1/R$. Here the *lower–upper solid (dashed) line* represents the BR for a single extra dimension without–with lepton KK modes, the real couplings being $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 50$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 80$ GeV) and $\rho = 0.01$

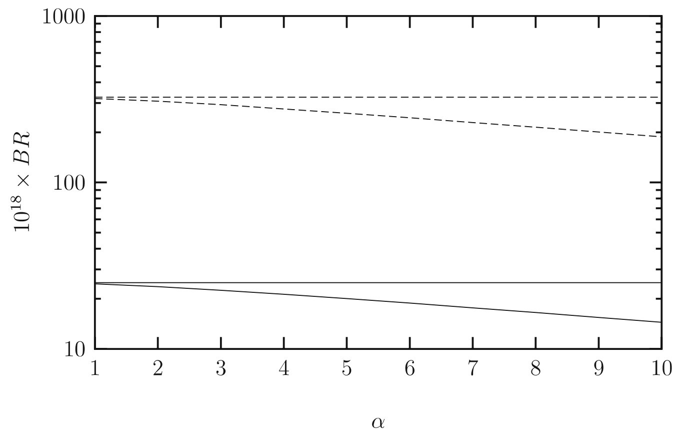

Fig. 9. The BR($\tau \to e\gamma$) with respect to α. Here the *lower–upper solid (dashed) line* represents the BR for a single extra dimension with lepton KK modes, for $y_{\mathrm{H}} = 0 - y_{\mathrm{H}} = \alpha\sigma$, the real couplings being $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.5$ GeV) and $R = 0.005\,\mathrm{GeV}^{-1}$

$\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.5$ GeV); $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 50$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 80$ GeV) and $\rho = 0.01$. These figures show that the enhancement of the BR with the addition of lepton KK modes is not so very sensitive to the compactification scale $1/R$ for its large values.

Now, we study the effects of the position of the localization point of the new Higgs doublet on the BR of the considered decays.

Figure 8 represents the BR($\mu \to e\gamma$) with respect to α, for different values of the Yukawa couplings $\bar{\xi}^{E}_{\mathrm{N},\tau\mu}$ and $\bar{\xi}^{E}_{\mathrm{N},\tau e}$. Here the lower–upper solid (dashed) line represents the BR for a single extra dimension with lepton KK modes, for $y_{\mathrm{H}} = 0 - y_{\mathrm{H}} = \alpha\sigma$, the real couplings being $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 50$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.5$ GeV) and $R = 0.005\,\mathrm{GeV}^{-1}$. This figure shows that the BR decreases with the increasing values of α, at the order of 45% in the interval $1 \le \alpha \le 10$, for large values of the Yukawa coupling.

In Figs. 9 and 10 we present the BR($\tau \to e\gamma$); BR($\tau \to \mu\gamma$) with respect to α. Here the lower–upper solid (dashed), line represents the BR for a single extra dimension with lepton KK modes, for $y_{\mathrm{H}} = 0 - y_{\mathrm{H}} = \alpha\sigma$, the real couplings being $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.1$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.5$ GeV); $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 100$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 50$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 80$ GeV) and $R = 0.005\,\mathrm{GeV}^{-1}$. These figures show that the BR decreases with the increasing values of α, at the order of 35%, 45% in the interval $1 \le \alpha \le 10$, for large values of the Yukawa coupling.

At this stage we would like to summarize our results.

– The BR is strongly sensitive to the Gaussian width of the localized neutral Higgs scalars and it increases with increasing values of the width for the decays under con-

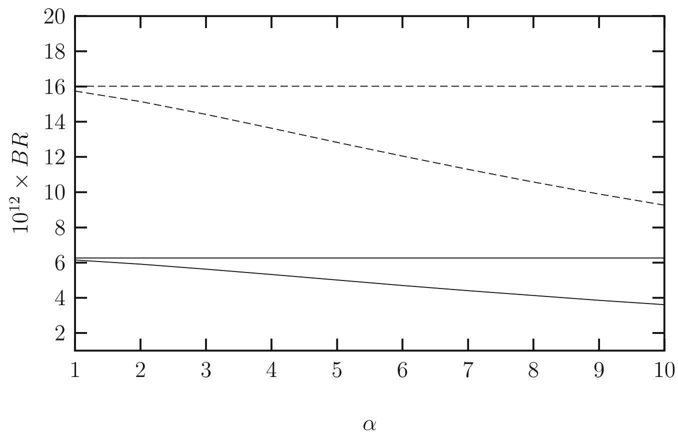

Fig. 10. The $\mathrm{BR}(\tau \to \mu\gamma)$ with respect to α. Here the *lower–upper solid (dashed) line* represents the BR for a single extra dimension with lepton KK modes, for $y_{\mathrm{H}} = 0 - y_{\mathrm{H}} = \alpha\sigma$, the real couplings being $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 50$ GeV ($\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 80$ GeV) and $R = 0.005\ \mathrm{GeV}^{-1}$

sideration. This enhancement is almost at the order of 10^4 in the range $0.005 \leq \rho \leq 0.05$. The inclusion of lepton KK modes brings additional enhancement at one order. The BR for $\mu \to e\gamma$ ($\tau \to e\gamma$, $\tau \to \mu\gamma$) is at most at the order of the magnitude of 10^{-12} (10^{-16}, 10^{-12}) for the intermediate values of the localization parameter ρ and the couplings taken.

– The BR decreases with the increasing distance, $y_{\mathrm{H}} = \alpha\sigma$, of the localization point of the new Higgs doublet from the origin in the extra dimension. With increasing values of α, there is almost 50% suppression of the BRs of the decays under consideration in the interval $1 \leq \alpha \leq 10$, for large values of the Yukawa couplings.

The improvement of the experimental results of the radiative LFV decay BRs would make it possible to search the effects of the extra dimension and the possible localization of the new Higgs bosons in the extra dimension.

Acknowledgements. I thank my father Oguz Iltan for his spiritual support.

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